5.3 and 5.6 Inverse and Inverse Trig Functions.

Review. Definition of Inverse Function

A function g is the **inverse** of the function f if:

A function can be written as a set of ordered pairs:

Function	**Inverse Function**
$f = \{(1,4),(2,5),(3,6),(4,7)\}$	$f^{-1} =$

Find the inverse of $f(x) = \sqrt{2x-3}$

Theorems.

1. The graph of f contains the point (a,b) iff the graph of f^{-1}contains the point

If f has an inverse, then

2. f is continuous on I $\rightarrow$ is continuous on its domain.

3. f is increasing (decr'g) on I $\rightarrow$ is increasing (decr'g) on its domain.

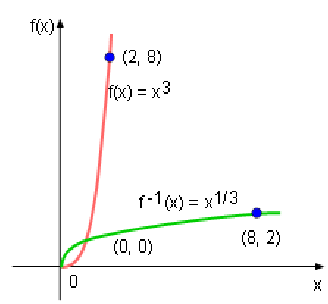
4. f is differentiable at c and f '(c) is not 0 $\rightarrow$ is diff'ble at f(c).

Derivative of an Inverse Function:

Let f^{-1} =g(x) and be the inverse of f , a differentiable function, then Graphically:

slope at of f at (2,8) is

slope of f^{-1} at (8,2) is



Numerically:

Analytically: Switch x and y and use implicit differentiation .

Written: The derivative of an inverse function at a point is the reciprocal of the derivative of original function at its corresponding point.

Ex. 1: Let $f(x) = x^3 - 2x^2 + 4x - 5$

What is the value of $f^{-1}(x)$ when $x = 3$

a) What is the value of $\left(f^{-1}\right)'(x)$ when $x = 3$

Ex. 2: Find the derivative of the inverse of $f(x) = x^5 + 7x^2$ **(switch x and y and use implicit)**

Ex. 3: Let $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is $g'(2)$?

Ex. 4: If $f(1)=-3$, $f'(1)=4$, and g is the inverse of f, then what is $g'(-3)$?

Practice Multiple choice. If $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is a differentiable and g is the inverse of f, then what is $g'(3)$?

a. $\frac{-1}{2}$ b. $\frac{-1}{8}$ c. $\frac{1}{6}$ d. $\frac{1}{3}$ e. cannot determine

Inverse Trigonometric Functions and Differentiation

Review

Ex. 1: Evaluate each of the following:

a) $\arcsin\left(-\frac{1}{2}\right)$ b) $\arccos(0)$ c) $\arctan\left(\sqrt{3}\right)$

Ex. 2: Use right triangles to evaluate the following expressions:

a) Given $y = \arcsin x$, find $\cos y$ b) Given $y = arc\sec\left(\frac{\sqrt{5}}{2}\right)$, find $\tan y$

Derivatives of Inverse Trigonometric Functions Let u be a differentiable function of x.

$\frac{d}{dx}[\arcsin u] =$ $\frac{d}{dx}[\arccos u] =$

$\frac{d}{dx}[\arctan u] =$ $\frac{d}{dx}[arc\cot u] =$

$\frac{d}{dx}[arc\sec u] =$ $\frac{d}{dx}[arc\csc u] =$

Ex. 3: Differentiate each of the following:

a) $\frac{d}{dx}[\arctan 3x]$ b) $\frac{d}{dx}\left[\arcsin\sqrt{x}\right]$ c) $\frac{d}{dx}\left[arc\sec e^{2x}\right]$

Ex. 4: Differentiate $y = \arcsin x + x\sqrt{1-x^2}$

Ex. 5: Write the equation of the tangent line to $y = e^{\arctan x}$ at $x = 1$.